The capacitor

In the early 1960's I pioneered the inter-connection of high speed (1nsec) logic gates at Motorola, Phoenix, Arixona (ref.25). One of the problems to be solved was the nature of the voltage decoupling at a point given by two parallel voltage planes. I asked Bill Herndon about the problem, and he gave me the answer: "It's a transmission line".[1] Bill learnt this from Stopper, whom I never met, who later worked for Burroughs (UNISYS) Corp. in Detroit.

The fact that parallel voltage planes, when entered at a point, present a resistive, not a reactive, impedance, was for me an important breakthrough. (It meant that as logic speeds increased, there would be no limitation presented by the problem of supplying +5v.) The reader should be able to grasp the reason why voltage plane decoupling is resistive by studying Figure 64, which shows the effect of a segment only of two planes as they are seen from a point.

During the next ten years, with the help of Dr. D. S. Walton, I gradually came to appreciate that, since a conventional capacitor was made up of two parallel voltage planes it also had a resistive, not a reactive (i.e. capacitive or inductive) source impedance when used to decouple the +5v supply to logic. Since the source impedance (= transmission line characteristic impedance) is well below one ohm, the transient current demand of logic gates approaching infinite speed can still be successfully satisfied with +5v from a standard capacitor of any type[2].

The capacitor is an energy store, and when energy is injected, it enters the capacitor *sideways* at the point where the two leads are joined to the capacitor. Nothing ever traverses a capacitor from one plate to the other[3]. This is clearly understood in the case of a transmission line. By definition, when a TEM wave travels down a transmission line, Figure 5, nothing travels sideways across the transmission line, or we would not have a transverse electromagnetic wave.

Comparison of the transmission line model with the lumped model of a capacitor in an RC circuit.

Consider a transmission line as shown in Figure 65 with

characteristic impedance Zo terminating in an open circuit. We will assume that R>>Zo.

When the switches are closed (at time t=0) a step of voltage $V\frac{Z_0}{R+Z_0}$ is propagated down the line. This

reflects from the open circuit at the right hand end to give a total voltage of $2V\frac{Z_0}{R+Z_0}$. Reflection from the

left hand end makes a further contribution of $V\frac{Z_0}{R+Z_0}\;\frac{R-Z_0}{R+Z_0}$ and so on. In general, after n two-way

passes the voltage is V_n and;

$$V_{n+1} = V_n + 2V\frac{Z_0}{R+Z_0}\left[\frac{R-Z_0}{R+Z_0}\right]^n \quad (1)$$

In order to avoid a rather difficult integration it is possible to sum the series to n terms using the formula

$$\sum V = a\frac{1-v^n}{1-v} \quad (2)$$

where a is the first term of a geometrical progression and v the ratio between terms. (This formula is easily verified by induction.) Substituting in (2) the parameters for (1),

$$a = 2V\frac{Z_0}{R+Z_0} \quad (3)$$

$$V = \frac{R-Z_0}{R+Z_0} \quad (4)$$

We obtain,

$$V_n = 2V \frac{\frac{Z_0}{R+Z_0}\left[1-\left\{\frac{R-Z_0}{R+Z_0}\right\}^n\right]}{1-\frac{R-Z_0}{R+Z_0}} \quad (5)$$

$$= V\left[1-\left\{\frac{R-Z_0}{R+Z_0}\right\}^n\right] \quad (6)$$

This is a correct description of what is happening as a capacitor charges. We can now go on to show that it is approximated by an exponential. We have

$$V_n = V\left[1-\left\{\frac{R-Z_0}{R+Z_0}\right\}^n\right] \quad (7)$$

Consider the term,

$$T = \left[\frac{R-Z_0}{R+Z_0}\right]^n .$$

.

$$= \left[\frac{1 - \frac{Z_0}{R}}{1 + \frac{Z_0}{R}} \right]^n .$$

If Zo/R<<1 this term is asymptotically equal to

$$T = \left[1 - \frac{2Z_0}{R} \right]^n .$$

Now define

$$k = 2Z_0 \frac{n}{R} .$$

Substitution gives

$$T = \left(1 - \frac{k}{n} \right)^n .$$

By definition, as $n \to \infty$ we have,

$$T = e^{-k} = e^{-\frac{2Z_0 n}{R}} .$$

and therefore:

$$V_n = V\left(1 - e^{-\frac{2Z_0 n}{R}} \right).$$

Now, after time t, $n = \frac{C}{2l}$, where C = velocity of propagation.

Thus,

$$V(t) = V\left[1 - e^{-\frac{Ct}{l}\frac{Z_0}{R}} \right].$$

For any transmission line it can be shown (p19) that

$$Z_0 = f\sqrt{\frac{\mu}{\varepsilon}},\ C = \frac{1}{\sqrt{\mu\varepsilon}},\ c = \frac{\varepsilon}{f}$$

where c= capacitance per unit length and f is a geometrical factor in each case. The "total capacitance" of length l

$$= lc = C.$$

Hence,

$$\frac{CZ_0}{lR} = \frac{1}{RC}$$

and therefore

$$V(t) = V\left[1 - e^{-\frac{t}{RC}}\right]$$

which is the standard result. This model does not require use of the concept of charge. A graphical comparison of the results is shown in Figure 66. [4]

Ref 25: Catt I., et al., A High Speed Integrated Circuit Scratchpad Memory, Fall Joint Computer Conference, Nov. 1966.

[1]ref.15, p40.

[2]Ref.3b, p216, refutes the fashionable nonsense about "RF capacitors".

[3]Similarly, the battery, p13, note 24, and the electrolyte.

[4]Calculations were by my co-author Dr. D.S. Walton. First published in Wireless World, dec78, p51.