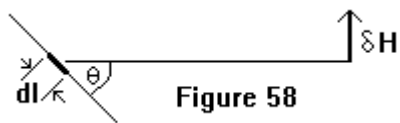


Calculation of Formulae.



Oersted discovered that a compass needle was deflected by a nearby electric current. This was quantified in the Biot-Savart Law, which says that if a small length δl of conductor carries current i ,

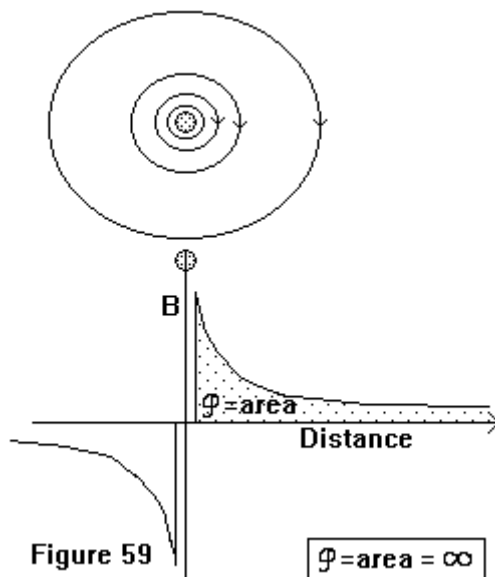
then the magnetic field strength at distance r and angle θ is

$$\delta H = \frac{i \delta l \sin \theta}{4 \pi r^2} .$$

(The $\sin \theta$ merely indicates that if the electric current is not in an optimum direction, then the field at that point is diminished.) This is restated as Ampere's Rule; $\int H dl = i$. This says in general that the line integral of the magnetic field in a closed loop equals the electric current passing through the loop^[1]. In particular, the magnetic field strength H in a circular path of radius r surrounding an electric current i at the centre is

$$H \cdot 2 \pi r = i, \text{ or } B = \frac{\mu i}{2 \pi r} .$$

Magnetic field surrounding current in a single long conductor. Self inductance of a long straight conductor.



At distance r out from the conductor,

$$B = \frac{\mu i}{2 \pi r} .$$

Integrate the total magnetic flux from the edge of the conductor out to infinity.

$$\begin{aligned} \phi &= \int Bdr \\ &= \frac{\mu i}{2\pi} \int_r^\infty \frac{dr}{r} \\ &= \frac{\mu i}{2\pi} [\log \infty - \log r] = \infty. \end{aligned}$$

The self inductance per unit length is

$$L = \frac{\phi}{i} = \infty.$$

The self inductance of a long straight conductor is infinite.

This is a recurrence of Kirchoff's First Law, that electric current cannot be sent from A to B. It can only be sent from A to B and back to A.

Self Inductance of a Pair of Parallel Conductors.

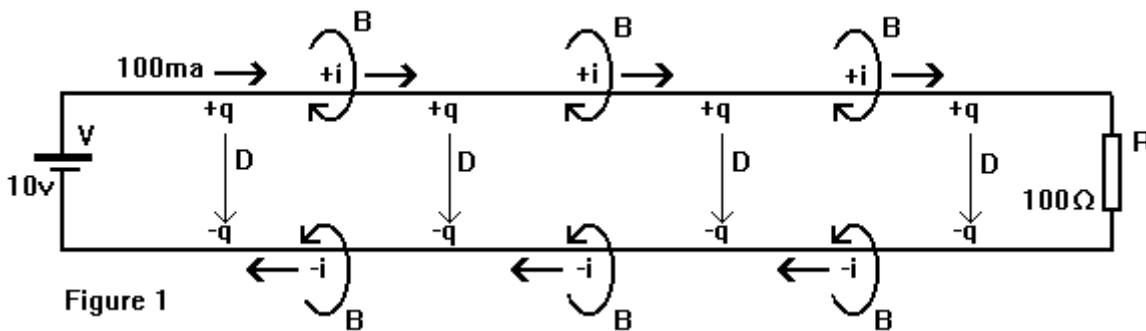


Figure 1

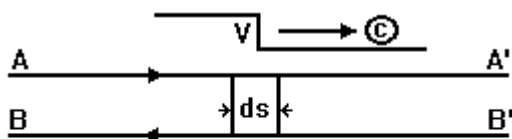


Figure 36

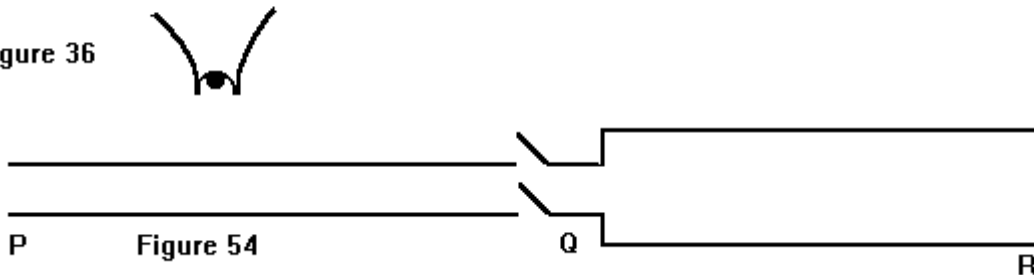


Figure 54

In Figures 1,36,54, the magnetic flux created by the forward current in the first wire is partially cancelled by the magnetic flux created by the current flowing in the opposite direction in the other wire. The result is a finite total flux and finite self inductance. To calculate this per unit length we integrate the magnetic flux passing

total flux and finite self inductance. To calculate this $l = \frac{\phi}{i}$ per unit length, we integrate the magnetic flux passing between the two wires. We integrate from r to a , the distance between wire centres^[2].

$$\begin{aligned}\phi &= \int Bdr \\ &= \frac{\mu i}{2\pi} \int_r^a \frac{dr}{r} \\ &= \frac{\mu i}{2\pi} (\log a - \log r) \\ &= \frac{\mu i}{2\pi} \log \frac{a}{r}\end{aligned}$$

This is the magnetic flux that one wire contributes. The total flux is twice this.

Therefore the self inductance per unit length (equal to $\frac{\phi}{i}$) is

$$2l = \frac{\mu}{\pi} \log \frac{a}{r}$$

Mutual Inductance between Two Pairs of Parallel Conductors.

The calculation is similar to before, except that the integration must be from the first conductor to the second conductor, that is, from b (=BP) to a (=BQ), for the effect of the nearer wire B. (b , a , follow the terminology in Table 1(g), p24. BP, BQ are in Fig.61.) Then we integrate similarly for the further wire A, between distances d (=AP) and c (=AQ). Subtract the two results, because, as we can see, the two fluxes are in opposite directions.

$$\begin{aligned}M &= \frac{\mu}{2\pi} \log \frac{a}{b} - \frac{\mu}{2\pi} \log \frac{c}{d} \\ \therefore M &= \frac{\mu}{2\pi} \log \frac{ad}{bc}.\end{aligned}$$

This is a general formula which remains true for any arrangement of two pairs of parallel wires (which do not have to lie in one plane). The practical case is more orderly. For instance, each pair might be similar. In that case, when the wires lie in one plane and the mean distance between pairs(=AP) is r times the separation(AB) of a pair, and $AP \gg AB$, the formula can be shown to reduce to

$$M = \frac{\mu}{2\pi} \log (1+r^2).$$

The mutual inductance is reversed if PQ is to the left of AB instead of to the right. However, the practical case for crosstalk in a printed circuit board is where the four similar wires (or actually, two wires plus their reflection

in a ground plane) form a rectangle instead lying in one plane. In this case, using the symbols in Table 1(f), p24,

$$M = \frac{\mu}{2\pi} \log \frac{a^2 + d^2}{d^2}.$$

In order to compare with the graphs for crosstalk in a PCB on p17, 18, we also need L_e , the self inductance for the even mode, and L_o , the self inductance for the odd mode. L_e is the flux which links a pair of wires AB as a result of current, half of it down A and back on B, and the other half down P and back on Q. This equals

$$\frac{L}{2} + \frac{M}{2}.$$

[The best formula to use for L in this context is

$$L = \frac{\mu}{2\pi} \log \frac{a^2}{r^2}$$

to make L and M look similar, rather than (p24) $\frac{\mu}{\pi} \log \frac{a}{r}$ as shown against Table 1(c).]

^[1]This is Faraday's discovery of electromagnetic induction. Changing magnetic flux through an area creates an electric voltage which tends to drive electric current around the perimeter in a direction opposing the change of flux. (The opposition is contained in Lenz's Law.)

^[2] W H Hayt, Engineering Electromagnetics, McGraw-Hill 1958,1989, p390/391, S. Ramo and others create a can of worms by not taking the centre of the two wires. Candidates are the near edge or the top point of one of the wires. I think this last choice leads them to end up with cosh rather than log, and create havoc in their wake. Since none of the choices is absolutely true, it is important and permissible to choose an approximation which leads to the most elegant, malleable and consistent set of equations. Experience shows that the proper choice is the centre of each of the wires. Absolute truth for the formula is approached for perfect conductors if the radius of the wires is insignificant compared with their separation.